Mathias Schacht

LECTURES

Monday:

We continue with the proof of the removal lemma for $K_4^{(3)}$, which implies $r_4(n) = o(n)$. For that we
– discuss of the hypergraph regularity lemma and its proof
– prove the so-called *dense counting lemma*. This lemma is one of the important components in the proof of the appropiate *counting lemma*.

Tuesday:

We reduce the proof of the *counting lemma* to the dense counting lemma, which completes the proof of $r_4(n) = o(n)$.
If time allows we discuss the extensions for general k.

DocCourse Prague 2005

MODERN METHODS IN RAMSEY THEORY

Programme coordinators:
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January 24 – February 25, 2005

MODERN METHODS *in* RAMSEY THEORY

PROGRAM FOR THE 5TH WEEK FEBRUARY 21 - FEBRUARY 25

Mo, Feb 21

09-10:30	*M. Schacht*	**S7**	Modern Methods in Ramsey Theory
10:40-12	*M. Schacht*	**S6**	Modern Methods in Ramsey Theory
14-17	*M. Schacht, Z. Dvořák*		Exercises

Tu, Feb 22

09-12	*M. Schacht* **Keiserstein Palace** **Malostranské nám. 23/37**		Modern Methods in Ramsey Theory
14-17	*M. Schacht, Z. Dvořák*		Exercises

We, Feb 23

55TH MATHEMATICAL COLLOQUIUM

10.40	*M. Simonovits*	**S7**	Turán Problems, Ramsey Problems, Simple and Random-looking Extremal Structures

Th, Feb 24

10.40	*V. T. Sós*	**S6**	A Hierarchy of Randomness for Graphs

Fr, Feb 25

10.40	*M. Ruszinko*	**S6**	Three-color Ramsey Numbers for Paths

Miklos Simonovits

Turán Problems, Ramsey Problems, Simple and Random-looking Extremal Structures

Turán and Ramsey type problems were always very closely related to each other.

For the original questions (i.e., Ramsey numbers and Turán numbers for complete graphs) the extremal graphs were simple looking for Turán type problems and very involved for the Ramsey case. When the selected excluded substructures were extended, the situation changed: Turán type problems with fuzzy extremal graphs and Ramsey extremal graphs with very regular, simple structures occurred.

In my talk I will analyse this strong relation through some specific examples. Among others, I will put emphasis on two basic cases, one of which is the problem of 3-coloring the edges of a complete graph and looking for monochromatic odd cycles:

Recently Skokan, Kohayakawa and myself improved the corresponding asymptotic result of Łuczak, proving an old conjecture of Bondy and Erdős, at least for n large:

We have proved that if $n > n_0$ and K_{4n-3} is 3-colored then it contains a monochromatic C_n (which is sharp for odd values of n. Some related results of Figaj, Łuczak, Gyárfás, Ruszinko, G. Sárközy, and Szemerédi will also be discussed.

The proof was obtained using the Regularity Lemma and the stability method. As I mentioned, the particular results will be embedded into discussing phenomena from the general setting.

Vera T. Sós

A Hierarchy of Randomness for Graphs

We formulate four families of problem with which we aim at distinguishing *different levels of randomness*.

The first one is completely non-random, being the ordinary Ramsey-Turán problem and in the subsequent three problems we formulate some randomized variations of it. These four levels form a hierarchy, the main topic of this work.

We formulate very briefly (and informally) the four questions for a special case. The questions are as follows:

Fix a family of graphs $\mathcal{L}$ and an integer $r \geq 2$.

(DD) How many edges guarantee for a graph G_n that if we r-color its edges arbitrarily, we *always* find a monochromatic $L \in \mathcal{L}$?

(DR) How many edges guarantee for a graph G_n that in *almost all* r-edge-colorings, we find a monochromatic $L \in \mathcal{L}$?

(RD) How many edges guarantee for a *random graph* R_n almost surely, that r-coloring its edges arbitrarily, we always find a monochromatic $L \in \mathcal{L}$?

(RR) How many edges guarantee for a random graph R_n *almost surely*, that r-coloring its edges at random, *almost all* the r-colorings contain a monochromatic $L \in \mathcal{L}$?

This is a joint work with M. Simonovits.

Miklos Ruszinko

Three-color Ramsey Numbers for Paths

For graphs $G_1, G_2, \ldots, G_r$, the Ramsey number $R(G_1, G_2, \ldots, G_r)$ is the smallest positive integer n such that if the edges of a complete graph K_n are partitioned into r disjoint color classes giving r graphs $H_1, H_2, \ldots, H_r$, then at least one H_i $(1 \leq i \leq r)$ has a subgraph isomorphic to G_i. The existence of such a positive integer is guaranteed by Ramsey's classical result. The number $R(G_1, G_2, \ldots, G_r)$ is called the Ramsey number for the graphs $G_1, G_2, \ldots, G_r$. There si very little known about the case when each G_i is a path P_n on n vertices. For $r = 2$ a well-known theorem of Gerencsér and Gyárfás states that

$$R(P_n, P_n) = \left\lfloor \frac{3n-2}{2} \right\rfloor.$$

For $r \geq 3$ the Ramsey numbers for P_n are not known.

We proof – for sufficiently large n – the following conjecture of Faudree and Schelp:

$$R(P_n, P_n, P_n) = \begin{cases} 2n-1 & \text{for odd } n, \\ 2n-2 & \text{for even } n, \end{cases}$$

for the three-color Ramsey numbers for paths on n vertices.

In an earlier version of this paper we proved the asymptotic result $R(P_n, P_n, P_n) = (2+o(1))n$. This was also obtained independently by Figaj and Łuczak. In the proof of the asymptotic result the main tools were the Regularity Lemma and the following lemma, suggested by Łuczak.

Lemma 1. *For every $\eta > 0$ there exists n_0 and $\varepsilon > 0$ such that for $n \geq n_0$ the following holds. For every 3-edge coloring of a graph G with n vertices and more than $(1-\varepsilon)\binom{n}{2}$ edges there exists a monochromatic connected matching covering at least $(1-\eta)n/2$ vertices of G.*

To obtain the exact result we sharpen Lemma 1. This is a joint result of András Gyárfás, Gábor Sárközy, Endre Szemerédi and myself.