DocCourse Prague 2005

MODERN METHODS IN RAMSEY THEORY

Programme coordinators:
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MODERN METHODS *in* RAMSEY THEORY

PROGRAM FOR THE 4TH WEEK FEBRUARY 14 - FEBRUARY 18

All lectures are held in S5 unless noted otherwise

Mo, Feb 14

09-12	*M. Schacht*	Modern Methods in Ramsey Theory
14-17	*M. Schacht, Z. Dvořák*	Exercises

Tu, Feb 15

09-12	*M. Schacht*	Modern Methods in Ramsey Theory
14-17	*M. Schacht, Z. Dvořák*	Exercises

Th, Feb 17

10	*J. Spencer*	Counting Connected Graphs Using Erdős Magic

Fr, Feb 18

10	*J. Cooper*	A Permutation Regularity Lemma

Mathias Schacht

Lectures

Monday:

The Counting Lemma.

Discussion of the Burr-Erdős Conjecture.

Blow Up Lemma.

Tuesday:

Hypergraph Removal Lemma and its connection to Szemerédi's Theorem.

Discussion of straightforward extensions of the Regularity Lemma to hypergraphs and its limitations in view of the Removal Lemma.

Discussion of a more refined approach to hypergraph regularity with a Counting Lemma.

Joel Spencer

Counting Connected Graphs Using Erdős Magic

Let $C(k,l)$ be the number of labelled connected graphs with k vertices, $k-1+l$ edges. We employ random graphs and breadth first search techniques to find the asymptotics of $C(k,l)$ whenever k and l tend to infinity. The probabilistic analysis has, at its heart, a biased bridge.

We place $k-1$ balls into k bins, ball j going into bin i with probability $p(1-p)^{i-1}/(1-p^k)$, a truncated exponential with "tilt" p. With Z_t balls in bin t the "queue walk" has $Y_0=1, Y_t=Y_{t-1}+Z_t-1$ and thus $Y_k=0$. When $p>k-3/2$ we analyze the probability that the walk has $Y_t>0$ for all $0<=t<k$.

We further use these techniques to analyze the joint distribution of the number of vertices and edges of the "giant component" of Erdős and Rényi. We further consider randomized algorithms that (often) efficiently generates a uniformly distributed connected graph with k vertices and $k-1+l$ edges.

Joshua Cooper

A Permutation Regularity Lemma

We introduce a permutation analogue of the celebrated Szemerédi Regularity Lemma, and derive a number of consequences. This tool allows us to provide a structural description of permutations which avoid a specified pattern, a result that permutations which scatter small intervals contain all possible patterns of a given size, a proof that every permutation avoiding a specified pattern has a nearly monotone linear-sized subset, and a "thin deletion" result. We also show how one can count sub-patterns of a permutation with an integral, and relate our results to permutation quasirandomness in a manner analogous to the graph-theoretic setting.